THIRD YEAR   B. Sc. MATHEMATICS

PAPER – I
REAL ANALYSIS

Duration: 3 Hours   **Max. Marks: 75**

UNIT - I

Real number system:

(i)   Field, ordered field, upper and lower bounds of a set in an ordered field. Superemum and infimum of a set and their properties. Completeness, Archimedean and denseness properties of an ordered field, the set Q of rational numbers as a non-complete dense. Archimedean ordered field and the set R of real numbers as a complete dense Archimedean ordered field.

(ii)   Open interval, closed interval, neighbourhood of a number. Real line R-Interior points and limit points of a set in R, open sets and closed sets in R and their properties, Nested Interval property. Bolzano-Weierstrass theorem, Heine Boral theorem, Compact set and connected set and their properties.

UNIT - II

(i)   Sequence, Bounded sequence, monotonic sequence, limit of a sequence, convergent sequence, properties of convergent sequence, Cauchy first and second theorems on limits, subsequence and it's properties, Cauchy sequence and it's properties, Cauchy general principle of convergence, Examples of convergent sequences.

(ii)   Series: Convergence and divergence of an Infinite series of real numbers, the necessary and sufficient conditions, various tests of convergence problems and their illustrations with regard to infinite series of positive terms. Series: Alternating series and Leibnitz test, absolute and semi (or conditional) convergence.

UNIT-III

Riemann Integration: Upper and Lower Darboux sum, Upper and Lower Riemann integrals, Riemann integrability of a bounded function in a closed interval, the necessary and sufficient condition for R integrability in terms of Darboux sums, properties of R-integrable functions, Fundamental theorem of integral Calculus.

UNIT - IV

(i)   Uniform convergence of sequences and series of functions, various tests including M_n-test and Weirstrass M-test, relations of uniform convergence with the continuity of the limit and the sum functions and also with term by term differentiation and term by term integration.

(ii)   Fourier series representation of periodic functions which are even, odd and none of these in the full interval or half the interval.

UNIT - V

(i)   Convergence of improper integrals - various tests and their applications, Evaluation of such integrals.

(ii)   Equivalent sets and their examples, nature of the relations of equivalence. Denumerable and non denumerable sets, countable and uncountable sets, Nature of subsets of a countable set and that of a denumerable (countable) sets, union of denumerable (countable) sets, Denumerability of the sets of integers and rational numbers and non denumerability of the closed unit interval [0, 1] and the sets of   real numbers and irrational numbers.

References:
1.   T. M. Apostol   : Mathematical Analysis.
2.   R. R. Goldbeg   : Real Analysis
3.   Walter Rudin   : Principles of Mathematical Analysis
4.   P.K. Jain& S. K. Kaushuik   : An introduction to Real Analysis.
5.   D. Somasundaram & B. Chaudhary   : A First Course of Mathematical Analysis.
6.   G. F. Simmon   : Introduction to Topology.
7.   Bhargava & Goyal   : Real Analysis.