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M.SC. (Mathematics) IIIrd SEMESTER EXAMINATION, 2019

Paper – IX

Tenser Analysis

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define contravariant tensors of first orders.

(ii) If A_{ij} is a skew symmetric tensor, then show that-

$$\left(\delta_j^i\,\delta_l^k + \delta_l^i\,\delta_j^k\right) A^{ik} = 0$$

(iii) Show that-

$$\frac{\partial g_{ik}}{\partial x^j} = [ij,k] + [kj,i]$$

(iv) Find the value of $\frac{\partial A_k}{\partial x^j}$

(v) Write the Euler's equation for calculus of variation.

(vi) Define Riemannian co-ordinates.

(vii) If R^{α}_{ijk} is cyclic symmetric, then calculate the value of $R^{\alpha}_{,ijk} + R^{\alpha}_{jki} + R^{\alpha}_{kij}$

(viii) Define Ricci-Tensor.

(ix) A potential field is given by $V = 3x^2y - yz$, then find the electric field at $P(2,-1,4)$.

(x) Find the value of graddiv A.

PART – B

UNIT –I

Q.2 Show that a skew symmetric tensor of rank two has $\frac{n(n-1)}{2}$ independent components in V_N.

Q.3 Prove that $A_{ij}\,B^i\,C^j$ is invariant if B^i and C^j are contravariant vectors and A_{ij} is a covariant tensor.

UNIT –II

Q.4 If the matrix of a V_N is such that $g_{ij} = 0$ for $i \neq j$ show that-

(i) $\begin{Bmatrix} i \\ j \quad k \end{Bmatrix} = 0$

(ii) $\begin{Bmatrix} i \\ j \quad j \end{Bmatrix} = -\frac{1}{2g_{ii}} \frac{\partial g_{jj}}{\partial x^i}$

(iii) $\begin{Bmatrix} i \\ i \quad j \end{Bmatrix} = \frac{\partial}{\partial x^j} \{\log \sqrt{g_{ii}}\}$

(iv) $\begin{Bmatrix} i \\ i \quad i \end{Bmatrix} = \frac{\partial}{\partial x^i} \{\log \sqrt{g_{ii}}\}$

Where *i,j* and *k* are not equal and the summation convention does not apply.

Q.5 Prove that-

(i) $g_{ij,k} = 0$

(ii) $g^{ij}_{,k} = 0$

(iii) $\delta^i_{j,k} = 0$

UNIT –III

Q.6 Show that at the pole P_0 of the geodesic co-ordinate system-

$$A_{i,jk} = \frac{\partial^2 A_i}{\partial x^j \partial x^k} - A_l \frac{\partial}{\partial x^k} \begin{Bmatrix} l \\ i \quad j \end{Bmatrix}$$

Q.7 Show that the parallel displacement of a vector taken all around a circle on the surface of a sphere does not lead back to the same vector except when the circle is a great circle that is a geodesic.

UNIT –IV

Q.8 The necessary and sufficient condition for a space V_N to be flat is that the Riemann-Christoffel tensor be identically zero i.e. $R^{\alpha}_{,ijk} = 0$

Q.9 Prove that $R_{1212} = -G\frac{\partial^2 G}{\partial u^2}$ for the v_2 whose line element is $ds^2 = du^2 + G^2 dv^2$ where G is a function of u and v.

UNIT –V

Q.10 State and proof Maxwell's equation in Tensor form.

Q.11 State and proof Einstein-Maxwell equation in general relativity.

PART – C / खण्ड – स

Q.12 Statement and proof of quotient law.

Q.13 If $[ij,k], \left\{ \begin{matrix} h \\ i \quad j \end{matrix} \right\}$ and $[\overline{ij,k}], \overline{\left\{ \begin{matrix} h \\ i \quad j \end{matrix} \right\}}$ are Christoffel symbols of first and second kind in co-ordinate system x^i and $\bar{x}^j$, then-

(a) $[\overline{ij,k}], = [uv, w] \frac{\partial x^u}{\partial \bar{x}^i} \frac{\partial x^v}{\partial \bar{x}^j} \frac{\partial x^w}{\partial \bar{x}^k} + g_{uv} \frac{\partial^2 x^u}{\partial \bar{x}^i \partial \bar{x}^j} \frac{\partial x^v}{\partial \bar{x}^k}$

(b) $\overline{\left\{ \begin{matrix} p \\ l \quad m \end{matrix} \right\}} = \left\{ \begin{matrix} s \\ i \quad j \end{matrix} \right\} \frac{\partial \bar{x}^p}{\partial x^s} \frac{\partial x^i}{\partial \bar{x}^l} \frac{\partial x^j}{\partial \bar{x}^m} + \frac{\partial \bar{x}^p}{\partial x^j} \frac{\partial^2 x^j}{\partial \bar{x}^l \partial \bar{x}^m}$

Q.14 A necessary and sufficient condition for vector B^i of variable magnitude to suffer a parallel displacement along a curve C is that-

$B^i_{,j} \frac{dx^j}{ds} = B^i f(s)$

Q.15 If the metric of a two dimensional flat space is $ds^2 = f(r)[(dx^1)^2 + (dx^2)^2]$

Show that $f(r) = C(r)^k$ where $r^2 = (dx^1)^2 + (dx^2)^2$ and C, k are constants.

Q.16 State and proof Lorentz invariance of Maxwell's equations.
