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M.SC. (MATHEMATICS) IIIrd SEMESTER EXAMINATION, 2019

Paper – VIII

Topology

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define Metric.

(ii) Define open and closed set.

(iii) Define base for a Topology.

(iv) What is First Countable?

(v) What is regular space?

(vi) Define Compact space.

(vii) Define Components of a space.

(viii) What is Locally Connected space?

(ix) Write statement of Stone – WEIERSTRASS theorem.

(x) Write statement of the Complex Stone – WEIERSTRASS theorem.

PART – B

UNIT –I

Q.2 Prove that the Inter section of a finite collection open set is open.

Q.3 Let f be a function from metric space X into a Metric space Y. Prove that f is continuous if and only if $f^{-1}(G)$ is open in X. Whenever G is open in Y.

UNIT –II

Q.4 A be subset of a topological space X, A' denote the set of all limit points of X, then with usual notation. Prove that- $\overline{A} = A \cup A'$.

Q.5 If X, Y, Z be topological space. If $f : X \to Y$ and $g : Y \to Z$ are continuous, then prove that $gof : X \to Z$ is also continuous.

UNIT –III

Q.6 Prove that every metric space in a Hausdorff space.

Q.7 Prove that every subspace of regular is regular.

UNIT –IV

Q.8 Prove that a subset of Real Line R is connected Iff, If it is an Interval. The space R is connected space.

Q.9 The component's of a totally disconnected space are it's point.

UNIT –V

Q.10 If f be a continuous real function defined in a closed interval [a, b], and let $\in > 0$ is given.

Prove that $\exists$ a polynomial p with real coefficients such that

$|f(x) - p(x)| < \in \ \forall \ x \in [a, b]$.

Q.11 State and prove the real stone Weierstrass theorem.

PART – C

Q.12 Prove that limit of a convergent sequence in a metric space is Unique.

Q.13 If f : X – Y be a bijection, then prove that the following properties of f are equivalent:

(a) f is homorphism.

(b) f is continuous and open.

(c) f is continuous and closed.

(d) $f(\overline{A}) = \overline{f(A)}$ for each $A \subset X$.

Q.14 Prove that every closed and bounded subset of R is compact.

Q.15 Prove that the finite product of locally connected spaces is locally connected.

Q.16 Prove that c [a, b] is a separable.
