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# 8227

## M.Sc. MATHEMATICS III$^{RD}$ SEMESTER
## EXAMINATION, 2019
## Paper - VII
# INTEGRAL EQUATIONS

Time: Three Hours

Maximum Marks: 80

***PART – A (खण्ड – अ)*** *[Marks: 20]*

*Answer all questions (50 words each).*

*All questions carry equal marks.*

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।*

*सभी प्रश्नों के अंक समान हैं।*

***PART – B (खण्ड – ब)*** *[Marks: 40]*

*Answer five questions (250 words each).*

*Selecting one from each unit. All questions carry equal marks.*

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।*

*सभी प्रश्नों के अंक समान हैं।*

***PART – C (खण्ड – स)*** *[Marks: 20]*

*Answer any two questions (300 words each).*

*All questions carry equal marks.*

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

*सभी प्रश्नों के अंक समान हैं।*

## PART – A

Q.1 (i) Define an integral equation with an example.

(ii) Define Volterra's integral equation of second kind.

(iii) Define separable Kernel of Fradholm integral equation of second kind with an example.

(iv) Define Eigen value and Eigen function of homogeneous Fredholm integral equation.

(v) Find resolvent kernel of the Fredholm integral equation when iterated kernel is given by –

$$k_m (x, t) = e^{x+t} \left(\frac{e^2 - 1}{2}\right)^{m-1}, m = 1, 2, 3, \ldots$$

(vi) Find the second iterated kernel of the kernel –

$k (x, t) = x – t, \quad a = 0, \quad b = 1$

(vii) Define symmetric kernel of the integral equation.

(viii) Define proper orthogonal set.

(ix) State the Hilbert-Schmidt Theorem.

(x) Define Volterra's integral equation with convolution type kernel with an example.

## PART – B

### UNIT –I

Q.2 Obtain Fredholm integral equation of second kind corresponding to the boundary value problem –

$$\frac{d^2 y}{d x^2} + \lambda y = x, \quad y (0) = 0, \quad y (1) = 1$$

Q.3 Prove that the homogeneous integral equation –

$$g (x) = \lambda \int_0^1 (3x - 2) g (t) dt$$

has no eigen values and eigen functions.

## UNIT –II

Q.4 Solve the homogeneous Fredholm integral equation of second kind with separable kernel.

$$g(x) = \lambda \int_0^{2\pi} \sin(x + t)\, g(t)\, dt$$

Q.5 Solve the Fredholm integral equation of second kind -

$$g(x) = x + \lambda \int_0^1 (x\,t^2 + x^2\,t)\, g(t)\, dt$$

## UNIT –III

Q.6 Using resolvent kernel method, solve –

$$g(x) = 1 + \lambda \int_0^1 e^{x-t}\, g(t)\, dt$$

Q.7 Using the method of successive approximations, solve the integral equation –

$$g(x) = 1 + \int_0^x (x - t)\, g(t)\, dt, \text{ taking } g_0(x) = 0.$$

## UNIT –IV

Q.8 If a kernel is symmetric, then prove that all its iterated kernels are also symmetric.

Q.9 Describe the expansion of Eigen functions and Bilinear form of symmetric kernels.

## UNIT –V

Q.10 Solve the symmetric integral equation –

$$g(x) = (x + 1)^2 + \int_{-1}^{1} (x\,t + x^2\,t^2)\, g(t)\, dt$$

by using Hilbert – Schmidt theorem.

Q.11 Convert the Volterra's integral equation with convolution type kernels –

$$g(x) = 1 + \int_0^x \sin(x - t)\, g(t)\, dt$$

in convolution form and hence solve it.

## PART – C

Q.12 Reduce the initial value problem –

$$\frac{d^2 y}{d x^2} - \sin x \frac{dy}{dx} + e^x y = x$$

with the initial condition y (0) = 1, y′ (0) = - 1 to a Volterra's integral equation of second kind. Conversely, derive the original differential equation with the initial conditions from the integral equation obtained.

Q.13 Solve the integral equation –

$$g(x) = \cos 3x + \lambda \int_0^{\pi} \cos(x+t)\, g(t)\, dt$$

Discuss all the cases.

Q.14 Find the resolvent kernel of the kernel –

$k(x, t) = \sin x \cos t, a = 0, \quad b = \frac{\pi}{2}$

Q.15 Prove that the Eigen functions of a symmetric kernel, corresponding to distinct eigenvalues are orthogonal.

Q.16 State the Fredholm's First Theorem and hence find the resolvent kernel of the kernel-

$k(x, t) = xe^t \quad ; \quad a = 0, \; b = 1$

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