

**8223**

**M.Sc. MATHEMATICS III<sup>rd</sup> SEMESTER  
EXAMINATION, 2019**

**Paper - III**

**DISCRETE MATHEMATICS-I**

Time: Three Hours

Maximum Marks: 80

**PART – A (खण्ड – अ)**

[Marks: 20]

*Answer all questions (50 words each).*

*All questions carry equal marks.*

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

**PART – B (खण्ड – ब)**

[Marks: 40]

*Answer five questions (250 words each).*

*Selecting one from each unit. All questions carry equal marks.*

प्रत्येक इकाई से एक-एक प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

**PART – C (खण्ड – स)**

[Marks: 20]

*Answer any two questions (300 words each).*

*All questions carry equal marks.*

कोई दो प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

## **PART – A**

- Q.1 (i) Define the statement, give one example.  
(ii) Define Monoids.  
(iii) Define Semi-homomorphism.  
(iv) Define direct product.  
(v) Define Lattice.  
(vi) Define Homomorphism.  
(vii) Define Boolean Lattice.  
(viii) Define Idempotent law in Boolean algebra.  
(ix) Define Conjunctive Normal Form.  
(x) Explain “AND” gate.

## **PART – B**

### **UNIT -I**

Q.2 Prove that following preposition are tautologies.

- (i)  $p \vee \sim p$   
(ii)  $\sim (p \wedge q) \vee q$   
(iii)  $p \Rightarrow (p \vee q)$

Q.3 A semi group  $(S, *)$  is a group IFF for  $a, b \in S$  each of the equations  $a * x = b$  and  $y * a = b$  has a solution in  $S$  for  $x$  and  $y$ .

### **UNIT -II**

Q.4 Prove that a Semi group has at most one absorbing element.

Q.5 Show that  $\exists$  a semi group homomorphism from semi group  $(\mathbb{N}, +)$  of natural number under addition to the semi group  $\{(0,1,2,3), +_4\}$ , where  $+_4$  denotes the operation of addition modulo 4 on the set  $(0, 1, 2, 3)$ .

### UNIT -III

Q.6 Let  $(L, \leq)$  be a Lattice, then for  $a, b, c, d \in L$ . Prove that :

(i)  $a \leq b \Rightarrow a \vee c \leq b \vee c$

(ii)  $a \leq b$  and  $c \leq d \Rightarrow a \vee c \leq b \vee d$

Q.7 Show the Lattice  $(L^3, \leq)$  of 3-tuples of 0 and 1 is complemented.

### UNIT -IV

Q.8 Prove that –

(i)  $(a + b)' = a'.b'$  (ii)  $(a. b)' = a' + b'$

Q.9 Express the Boolean function  $f(x, y, z) = x + y'z$  in a sum of minterm.

### UNIT -V

Q.10 Find the logic Networks corresponding to Boolean expression.

(i)  $AB + CD$  (ii)  $X' Y' Z + X' YZ + XY'$

Q.11 Prove the Boolean Identify –

$$A \oplus B \oplus A.B = A + B$$

### PART - C

Q.12 Construct the truth table for-

(i)  $p \vee \sim q = p$

(ii)  $(\sim(p \wedge q) \vee r) = \sim p$

Q.13 Let  $(S, *)$ ,  $(V, 0)$  be semi group  $f : S \rightarrow T$  and  $g : T \rightarrow V$  be a semi group homomorphism.

Then prove that  $g \circ f : S \rightarrow V$  is a semi group homomorphism from  $(S, *)$  to  $(V, 0)$ .

Q.14 Show that the dual of a Complemented Lattice is also Complemented Lattice.

Q.15 Prove that in any Boolean algebra order relation  $\leq$  is partial order relation.

Q.16 Find Karnaugh map and simplify the expressions:

(i)  $AB' + A'B'$

(ii)  $AB' + A'B$

(iii)  $AB' + A'B + A'B'$

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