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M.Sc. IInd Semester EXAMINATION, 2018

MATHEMATICS

Paper – V

(Differential Geometry-II)

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer five questions (**250** words each).*

*Selecting **one** from each unit. All questions carry equal marks.*

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो** प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

1. Solve all questions:

(i) Define tangent line to a curve.

(ii) Write serret frenet formulas.

(iii) Define a osculating sphere.

(iv) Define edge of regression of a system of surface.

(v) Define a developable surface and write the condition that the surface $x = az + \alpha$, $y = bz + \beta$ to be developable.

(vi) Write the formula for curvature of normal section in terms of fundamental magnitude.

(vii) Write the expression for radius of curvature of a given section through any point of the surface $z = f(x, y)$.

(viii) Write the determinant form of the differential equation to the projection of two lines of curvature.

(ix) Write the differential equation to find Principal radii of the surface.

(x) Define linear element of a surface.

PART – B

UNIT –I

2. Find the lines that have four point contact at (0, 0, 1) with the surface.
$x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z = 1.$

OR

3. Find the radius of curvature and torsion of the helix $x = a\cos\theta$, $y = a\sin\theta$, $z = a\theta\tan\alpha$.

UNIT –II

4. If the tangent to a curve makes a constant angle α with a fixed line then prove $\sigma = \pm\rho\tan\alpha$.

OR

5. Find the envelope of the plane

$$\frac{x}{a}\cos\theta\ \sin\psi + \frac{y}{b}\sin\theta\ \sin\psi + \frac{z}{c}\ \cos\psi = 1$$

UNIT –III

6. Find the equation to the developable surface which has the following curve for their edge of regression $x = 6t$, $y = 3t^2$, $z = 2t^3$.

OR

7. Find the curvature of the normal section of the hellicoid

$x = u \cos\theta,\ y = u \sin\theta,\ z = f(u) + c\theta$

UNIT –IV

8. For the hyperbolic paraboloid $2x = 7x^2 + 6xy - y^2$. Prove that the principal radii at the origion $\frac{1}{8}$ and $\frac{-1}{2}$ and Principal sections are $x = 3y$, $3x = -y$.

OR

9. If ℓ_1, m_1, n_1 are direction cosines of the tangent to the line of curvature and ℓ, m, n are direction cosines of the normal to the surface at the point then prove $\frac{d\ell}{\ell_1} = \frac{dm}{m_1} = \frac{dy}{n_1}$.

UNIT –V

10. Find the vertices of the ellipsoid $\frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} = 1$.

OR

11. For the surface $\frac{x}{a} = \frac{u+v}{2}$, $\frac{y}{b} = \frac{u+v}{2}$, $z = \frac{uv}{2}$. Prove that the principal radii are given by $a^2 b^2 \rho^2 + \lambda a b \rho (a^2 - b^2 + u v) - \lambda^4 = 0$ where

$4\lambda^2 = 4a^2b^2 + a^2(u-v)^2 + b^2(u+v)^2$

PART – C

12. Prove that the points of the curve of intersection of the sphere and conicoid $rx^2 + ry^2 + rz^2 = 1,\ \ ax^2 + by^2 + cz^2 = 1$ at which the osculating plane pass through the origin lies on the cone $\frac{a-r}{b-c} x^4 + \frac{b-r}{c-a} y^4 + \frac{c-r}{a-b} z^4 = 0$

13. Find the envelope of the plane $3xt^2 - 3yt + z = t^3$ and show that its edge of regression is the curve of intersection of the surface $y^2 = xz$, $xy = z$.

14. Show that the developable which passes through the curves $z = 0$, $y^2 = 4ax$; $x = 0$, $y^2 = 4bz$ is the cylinder $y^2 = 4ax + 4bz$.

15. For the surface $x = u \cos \theta$, $y = u \sin\theta$, $z = f(\theta)$ prove that the angles that the lines of curvature make with the angle are given by

$\tan^2 \alpha + \frac{f''}{f'} \frac{u}{\sqrt{u^2 + f^{12}}} \tan \alpha - 1 = 0$ Where dash denote differentiate wr to θ.

16. Prove that for the surface $x = 3u(1+ \nu^2) - u^3$, $y = 3u(1 + u^2) - u^3$, $z = 3(u^2 - \nu^2)$, the principal radii at any point are $\pm\frac{3}{2}(1+u^2+\nu^2)^2$ and the lines of curvature are given by $u = c_1$, $\nu = c_2$ where c_1 and c_2 are arbitrary constants.
