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M.Sc. IInd Semester EXAMINATION, 2018

MATHEMATICS

Paper – III

(Special Functions)

Time: Three Hours
Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer five questions (**250** words each)selecting*

***one** from each unit. All questions carry equal marks.*

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो** प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (a) Define regular singular point of the equation

$$P(x)\frac{d^2y}{dx^2}+Q(x)\frac{dy}{dx}+R(x)y=0$$

(b) What do you mean by Frobenius Method?

(c) Write the generating function of $P_n(x)$.

(d) What is a Legendre's polynomial?

(e) Write the associated Legendre equation.

(f) Calculate the value of $P_n(1)$ from generating function.

(g) What is a Bessel Function?

(h) From $\int_0^x x^{n+1}J_n(x)dx = x^{n+1}J_{n+1}(x), n>-1$

deduce $J_0'(x)=-J_1(x)$

(i) Write the values of $H_1(x)$ & $H_2(x)$.

(j) Define Orthogonal Polynomials.

PART – B

UNIT –I

Q.2 Solve Legendre's equation-

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0 \text{ at an ordinary point.}$$

Q.3 If $\gamma>\beta>0$ and $|x|<1$, then prove that-

$$2F_1(\alpha,\beta;\gamma;x)\frac{\Gamma\gamma}{\Gamma\beta\,\Gamma\gamma-\beta}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tx)^{-\alpha}dt$$

UNIT –II

Q.4 Establish Rodrigues formula for $P_n(x)$.

Q.5 Prove that-

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & ;\text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$

UNIT –III

Q.6 Prove that-

$${}^{n}P_n(x) = x\,P'_n(x) - P'_{n-1}(x)$$

Q.7 Show that zeros of the Legendre Polynomials $P_n(x)$ are all real and lies between -1 and 1.

UNIT –IV

Q.8 Show that-

$$xJ'_n(x) = nJ_n(x) - xJ_{n+1}(x)$$

Q.9 Establish-

$$2nJ_n(x) = x\left[J_{n-1}(x) + J_{n+1}(x)\right]$$

UNIT –V

Q.10 Prove that-

$$P_n(x) = \frac{2}{\underline{\lfloor n}\,\sqrt{\pi}} \int_0^\infty e^{-t^2}.t^n\,H_n(xt)\,dt$$

Q.11 Show that-

$$\int_0^\infty e^{-x}\,L_n(x)L_m(x)dx = \delta_{mn}, \text{ where}$$

δ_{mn} is the kronecker delta.

PART – C

Q.12 Solve Bessel's equation-

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-n^2\right)y=0 \text{ ,when}$$

m is not an integer (general solution).

Q.13 Establish the following theorems for hypergeometric functions

(a) Gauss's Theorem

(b) Kummer's Theorem

Q.14 Prove that-

$$\int_{-1}^{1} P_m^k(x)\,P_l^k(x)\,dx=(-1)^k\,\frac{\lfloor k+1}{\lfloor 1-k}\left(\frac{2}{2l+1}\right)\delta_{lm},$$

where δ_{lm} is the kronecker delta.

Q.15 If n is a positive integer, then prove that-

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos\,(n\phi - x\sin\phi)\,d\phi$$

Also, show that this result holds for integer n.

Q.16 Establish the formula-

$$L_n(x)=\frac{e^x}{\lfloor n}\,\frac{d^n}{dx^n}\left(x^n e^{-x}\right)$$

and obtain the values of $L_n(n)$ for n= 0,1,2 and 3.
