Roll No. .... Total Pages: 04

7222

M.Sc. IInd Semester EXAMINATION, 2018

MATHEMATICS

Paper – II

(Complex Analysis)

Time: Three Hours
Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer five questions (**250** words each).*

*Selecting **one** from each unit. All questions carry equal marks.*

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो** प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define Harmonic function and show that $u = y^3 - 3x^2 y$ is Harmonic.

(ii) Define radius of convergence of power series.

(iii) Define cross ratio

(iv) Explain bilinear transformation

(v) Define multi connected region

(vi) Write Cauchy's integral formula of successes derivative of an analytic function.

(vii) State Taylor's theorem

(viii) Write Poisson's integral formula

(ix) Define isolated and non isolated singularity with example.

(x) Find the residue of $\frac{z^2}{z^2 + a^2}$ at $z = ia$

PART – B

UNIT –I

Q.2 If $u = x^3 - 3x\,y^2$, show that there exist a function v (x,y) such that w = u + iv is analytic in a finite region.

Q.3 Find the domain of convergence of the power series $\left(\frac{iz-1}{z+i}\right)^n$

UNIT –II

Q.4 Find the bilinear transformation that maps the points $z = \infty, z_2 = i$ and $z_3 = 0$ into the points $w_1 = 0, w_2 = c$ and $w_3 = \infty$

Q.5 Show that the bilinear transformation of any two points which are inverse with respect to a circle into two points are inverse with respect to the transformed circle.

UNIT –III

Q.6 Using the definition of the integral of $f(z)$ on a given path, evaluate $\int_{-2+i}^{5+3i} z^3 dz$

Q.7 State and prove Cauchy's theorem

UNIT –IV

Q.8 If a function $f(z)$ is analytic for all finite values of z and is bounded then prove it is a constant function.

Q.9 State and prove Morea's theorem.

UNIT –V

Q.10 What kind of singularity have the function

$$f(z) = \frac{1}{1-e^z} \text{ at } z = 2\pi i \text{ and } f(z) = e^z \text{ at } z = \infty$$

Q.11 Find the residue of the function $\frac{\cot \pi z}{(z-a)^2}$

PART – C

Q.12 If $f(z) = n + iv$ is an analytic function of

$$z = x + iy \text{ and } n - v = \frac{e^{y} - \cos x + \sin x}{\cos hy - \cos x} \text{ find } f(z)$$

subject to the condition $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$

Q.13 Find the invariant points and normal form of the bilinear transformation

$$w = \frac{3z-4}{z-1} \text{ and } w = \frac{z}{z-2}$$

Q.14 Derive Cauchy's integral formula for the derivative

Q.15 Find the Taylor's or Laurent's series which represent the function $\frac{1}{(1+z^2)\ (z+2)}$

(i) when $| z | < 1$

(ii) when $l < | z | < 2$

(iii) when $| z | > 2$

Q.16 (i) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at z = 1, 2, 3, and infinity and show that their sum is zero.

(ii) Evaluate $\int_C \frac{e^2 d^2}{z(z-1)^2}$ where c is the circle $|z| = 2$
