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M.Sc. IVth SEMESTER EXAMINATION, 2019

MATHEMATICS

Paper – IVth

DSE – 04 [Optimization Techniques]

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 Answer all questions -

(i) Define unconstrained problems of maxima and minima.

(ii) Obtain the set of necessary conditions for the non-linear programming problem:

Maximize $Z = x_1^2 + 3x_2^2 + 5x_3^2$

Subject to the constraints:

$x_1 + x_2 + 3x_3 = 2,\ 5x_1 + 2x_2 + x_3 = 5$ and $x_1, x_2, x_3 \geq 0$

(iii) Explain Saddle Point Problems.

(iv) Write Kuhn-Tucker necessary and sufficient conditions.

(v) What is quadratic programming problem?

(vi) Use Beale's method for solving the quadratic programming problem (only one step)-

Max. $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to : $x_1 + 2x_2 \leq 2$ and $x_1, x_2 \geq 0$

(vii) Explain a dynamic programming problem.

(viii) State a sufficient condition for a two-stage optimization problem to be solved by dynamic programming.

(ix) Define maximum flow algorithm.

(x) Define Shortest Route Problem.

PART – B

UNIT – I

Q.2 Find the maximum or minimum of the function -

$f(x) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56$

Q.3 Obtain the necessary and sufficient conditions for the optimum solution of the following non-linear programming problems:

Min. $Z = f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to the constraints: $x_1 + x_2 = 7$ and $x_1, x_2 \geq 0$

UNIT – II

Q.4 Write the Kuhn-Tucker conditions for the following minimization problem:

Minimize. $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to : $g_1(x) = 2x_1 + x_2 \leq 5$

$g_2(x) = x_1 + x_3 \leq 2$

$g_3(x) = -x_1 \leq -1$

$g_4(x) = -x_2 \leq -2$

$g_5(x) = -x_3 \leq 0$

Q.5 Let $x \in R^n$, $u \in R^m$ and f be a function of x and u for a point (x*, u*) to be a non-negative saddle point of f (x, u) it is necessary that -

(i) $f_{x^*} \leq 0,\ f_{x^*}^T\ x^* = 0$, (ii) $f_{u^*} \geq 0, f_{u^*}^T u^* = 0$, for $x^* \geq 0, u^* \geq 0$

UNIT – III

Q.6 Solve the quadratic programming problem by Wolfe's method:

Max. Z = $2x_1 + x_2 - x_1^2$,

Subject to : $2x_1 + 3x_2 \leq 6$, $2x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$.

Q.7 Solve the quadratic programming problem by Beale's method:

Max. Z = $10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to : $x_1 + 2x_2 + x_3 = 10$, $x_1 + x_2 + x_3 = 9$.

and $x_1, x_2, x_3 \geq 0$.

UNIT – IV

Q.8 Use dynamic programming to show that :

$\sum_{i=1}^{n} Pi \log Pi$ Subject to $\sum_{i=1}^{n} Pi = 1, Pi \geq 0$ is minimum when $P_1 = P_2 = \ldots\ldots = P_n = 1/n$.

Q.9 Formulate the following problem as a multi stage problem and then solve it :

Min. Z = $x_1^2 + x_2^2 + \ldots + x_n^2$

Subject to $x_1 . x_2 . x_3 \ldots\ldots\ldots x_n = b$

and $x_1, x_2 \ldots\ldots, x_n \geq 0$

UNIT –V

Q.10 Draw the network (N, L) where N and L are given by N = {1, 2, 3, 4, 5, 6} and L = {1 - 2, 1 - 5, 2 - 3, 2 - 4, 3 - 5, 3 - 4, 4 - 3, 4 - 6, 5 - 2, 5 - 6} construct a spanning tree for the network.

Q.11 Consider the transportation problem:

Source	Destination			Supply
	1	2	3	
1	6	7	4	40
2	5	8	6	60
Demand	30	40	30	

Formulate the network representation of this problem as a minimum cost flow problem.

PART – C

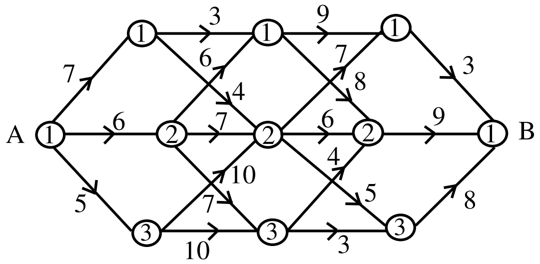
Q.12 A necessary condition for a continuous function f(x) with continuous first and second partial derivatives to have an extreme point at x_0 is that each first partial derivative of f(x). Evaluated at x_0 vanish, that is $\nabla f(x_0) = 0$.

Q.13 State and proof Kuhn – Tucker necessary conditions.

Q.14 State and proof Wolfe's method for DPP.

Q.15 Find the shortest path from vertex A to vertex B along axes joining various vertices lying between A and B. Length of each path is given.

J = 0 J = 1 J = 2 J = 3 J = 4



Q.16 Use Dijkstra's algorithm to determine a shortest path from A to C for the following network.