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M.Sc. IVth SEMESTER EXAMINATION, 2019

MATHEMATICS

Paper – I

Core Course-13 Functional Analysis

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (a) Define the norm Linear spaces.

(b) State the Riesz- lemma.

(c) Define the Banach spaces.

(d) State the Hahn- Banach theorem.

(e) Define the Inner product space.

(f) If x and y are any two vectors in a Hilbert space, then

$\|x + y\|^2 + \|x - y\|^2 = 2 \| x \|^2 + 2 \| y \|^2$

(g) Define the complete orthonormal set.

(h) Define the conjugate space.

(i) Define the Self-adjoint operator.

(j) Define the unitary operator.

PART – B

UNIT –I

Q.2 State and prove the Minkowski inequality.

Q.3 Let N and N′ be Normed linear spaces over the same scalar field and let T be a linear transformation of N into N′. Then T is bounded iff it is continuous.

UNIT –II

Q.4 State and prove the open mapping theorem.

Q.5 State and prove the closed graph theorem.

UNIT –III

Q.6 If x and y are any two vectors in a Hilbert space H, then $|(x, y)| \leq \|x\| \|y\|$.

Q.7 Let S be a non - empty subset of a Hilbert space H. Then $S^{\perp}$ is a closed linear subspace of H.

UNIT –IV

Q.8 Show that in a Hilbert space H an orthonormal set S is complete if and only if $x \perp s \Rightarrow x = 0$.

Q.9 Show that the mapping $\psi : H \to H^*$ define by $\psi(y) = fy$ where $fy(x) = (x, y)\ \forall\ x \in H$ is one- one, onto but not linear and an isometry.

UNIT –V

Q.10 Show that -

(i) $\|T^* T\| = \|T\|^2$

(ii) $(T_1 T_2)^* = T_2^* T_1^*$

Q.11 If N is a Normal operator on a Hilbert space H, then $\|N^2\| = \|N\|^2$

PART – C

Q.12 Show that the linear space R^n and C^n of all n- tuples $x = (x_1, x_2 \ldots\ldots\ldots., x_n)$ of real and complex numbers are Banach spaces under the norm.

$$\|x\| = \left(\sum_{i=1}^{n} |xi|^2 \right)^{1/2}$$

Q.13 State and prove the uniform bounded theorem.

Q.14 If M is a closed linear subspace of a Hilbert space H, then $H = M \oplus M^{\perp}$.

Q.15 If {ei} is an orthonormal set in a Hilbert space H and if x, y are arbitrary vectors in H, then, $\sum |(x, ei)\overline{(y, ei)}| \leq \|x\| \, \|y\|$.

Q.16 An operator T on a Hilbert space H is unitary iff it is an isometric isomorphism of H onto itself.
