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M.Sc. MATHEMATICS Ist SEMESTER EXAMINATION, 2019

Paper – II

REAL ANALYSIS

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) If A is a singleton set then prove that $m^*(A) = 0$

(ii) Define outer measure of any subset of R.

(iii) Define signed measure on a measurable space (X,B).

(iv) Define G_δ and F_σ sets.

(v) Show that constant function with measurable domain is measurable.

(vi) State 'almost everywhere' property of a measurable set.

(vii) Define step function with example.

(viii) Define uniform convergence of measurable function.

(ix) State bounded convergence theorem.

(x) Give an example of a bounded and measurable function which is not Reimann integrable but which is Lebesgue integrable.

PART – B

UNIT –I

Q.2 If E_1 and E_2 are disjoint measurable sets then prove that –

$$m\left(\bigcup_{K=1}^{\infty} E_K\right) = \sum_{K=1}^{\infty} m(E_K)$$

Q.3 Let A be any subset of R, then for every $x \in R$, Prove that $m^*(A+x) = m^*(A)$.

UNIT –II

Q.4 Prove that the family M of all measurable sets is a σ- Algebra.

Q.5 For a set E, prove that the following statement are equivalent

(i) E is measurable

(ii) Given $\varepsilon > 0$, $\exists$ an open set $O \supset E$ such that $m^*\left(\frac{O}{E}\right) < \epsilon$

(iii) There is a G_δ set $G \supset E$ such that $m^*\left(\frac{G}{E}\right) = 0$

UNIT –III

Q.6 If f is a measurable function defined on a measurable set E, then prove that the set $\{x : f(x) = \alpha\}$ is measurable for such extended real number α.

Q.7 Prove that a continuous function defined on measureable set is measurable. Is the converse true or not? Justify.

UNIT –IV

Q.8 If a sequence {fn} converges in measure to a function f, then prove that limit function f is unique almost everywhere.

Q.9 If the sequence of function {fn} converges in measure of two functions f(x) and g(x), then these limit functions are equivalent.

UNIT –V

Q.10 Let f and g be bounded measurable functions defined on a set E of finite measure. Let f = g almost everywhere then prove that:

$\int_{\in} f = \int_{\in} g$ Is converse true? Verify.

Q.11 If f is a measurable function on a measurable set E and if $a \leq f(x) \geq b$ then prove

$$a.m(E) \leq \int_{\in} f(x)\,dx \leq b.m(E)$$

PART – C

Q.12 If I is any interval then prove that

$$m^*(I) = \ell(I)$$

Q.13 Prove that there exist a non- measurable set in the interval (0, 1).

Q.14 If a sequence {fn} of measurable functions defined on a measurable set E converge point wise to a function f on E then prove that f is measurable.

Q.15 Let E be a measurable set with $m(E) < \infty$ and {fn} be a sequence of measurable function which converge to f a.e on E. then given $r > 0$ there exists a set $A \subset E$ with $m(A) < r$ such that the sequence {fn} converges to f uniformly on $\frac{E}{A}$.

Q.16 State and prove Lebesgue convergence theorem.
