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M.SC. MATHEMATICS IST SEMESTER EXAMINATION, 2019

Paper – I

ALGEBRA

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define external direct product.

(ii) Explain commentator sub group.

(iii) Define composition series.

(iv) Let G be a group of order 15, then find the number of 3-sylow subgroup of G.

(v) Define abelian group with example.

(vi) Explain solvable group with example.

(vii) Define Projection.

(viii) Explain Annihilator and also write the formula for calculating its dimension.

(ix) Define Diagonalization of a linear operator.

(x) Define Quadratic forms.

PART – B

UNIT –I

Q.2 Let G be a group and G be the internal direct product of two of its subgroups H_1 and H_2 then H_1 and H_2 are normal subgroup of G and

$$\frac{G}{H_1} \cong H_2 \text{ and } \frac{G}{H_2} \cong H_1.$$

Q.3 Let G' be the commentator subgroup of a group G. Then G is abelian if and only if $G' = \{e\}$, e being the identity element of G.

UNIT –II

Q.4 State and prove Sylow's third theorem.

Q.5 Show that no group of order 108 is simple.

UNIT –III

Q.6 Prove that every nilpotent is solvable but converse is not true.

Q.7 A group G is solvable if and only if $G^{(R)} = (e)$ for some non-negative integer R.

UNIT –IV

Q.8 Let E be a linear transformation, then E is a projection $\Leftrightarrow$ $(I - E)$ is a projection.

Q.9 If W_1 is T-invariant on V(F), then for every projection E on W_1, we have ETE = TE and conversely.

UNIT –V

Q.10 If the field F of characteristic $\neq 2$, then every symmetric bilinear forms on V(F) is uniquely determined by the corresponding quadratic form.

Q.11 A linear transformation A on a finite dimensional vector spaces is invertible if and only if it is non-singular.

PART – C

Q.12 (a) State and prove Cauchy's theorem for finite abelian group.

(b) If G_1 and G_2 are groups, then the subsets $G_1 \times \{e_2\}$ and $\{e_1\} \times G_2$ are normal subgroups of $G_1 \times G_2$ and is isomorphic to G_1 and G_2 respectively.

Q.13 State and prove Jordan-Holder theorem for finite group.

Q.14 State and prove Fundamental theorem for finite abelian groups.

Q.15 Let W be a subspace of V(F), then dim A(W) = dim V- dim W.

Q.16 Let T be a linear operator on $R^3(R)$ which is represented in the standard ordered basis by the matrix.

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Prove that T is diagonalizable.
